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Pre Calculus 12 Honors Ch4 Review Trigonometry

1. Solve the following for $0° \le \theta \le 360°$. Show all your work and steps (5marks each)

i) $84\tan^2 2\theta - 73\tan 2\theta - 45 = 0$

$\tan 2\theta = A$

$84A^2 - 73A - 45 = 0$

$A = \frac{73 \pm \sqrt{73^2 + 4(84)(45)}}{2(84)} = \frac{9}{7}$ or $-\frac{5}{12}$

$\tan 2\theta = \frac{9}{7}$

$2\theta = \tan^{-1}\left(\frac{9}{7}\right)$

$= 52.1°, 412.1°, 232.1°, 592.1°$

$\theta = 26.1°, 206.1°, 116.1°, 296.1°$

$\tan 2\theta = -\frac{5}{12}$

$2\theta = -22.6°$ or $202.6°$ or $-382.6°$ or $562.6°$

$\theta = 348.7°, 101.3°, 168.7°, 281.3°$

ii) $20\csc^2 3\theta + 13\csc 3\theta - 72 = 0$

$\csc 3\theta = A$

$20A^2 + 13A - 72 = 0$

$A_1 = -\frac{8}{5}$ $A_2 = -\frac{9}{4}$

$\frac{1}{\sin 3\theta} = \frac{8}{5} \Rightarrow \sin 3\theta = \frac{5}{8}$ $\sin 3\theta = -\frac{4}{9}$

$3\theta = \sin^{-1}\left(\frac{5}{8}\right) = 38.68°, 398.6°, 758.6°$

$321.3°, 681.3°, 1041.3°$

$\theta = 12.89°, 132.9°, 244.3°, 107.1°, 227.1°, 347.$

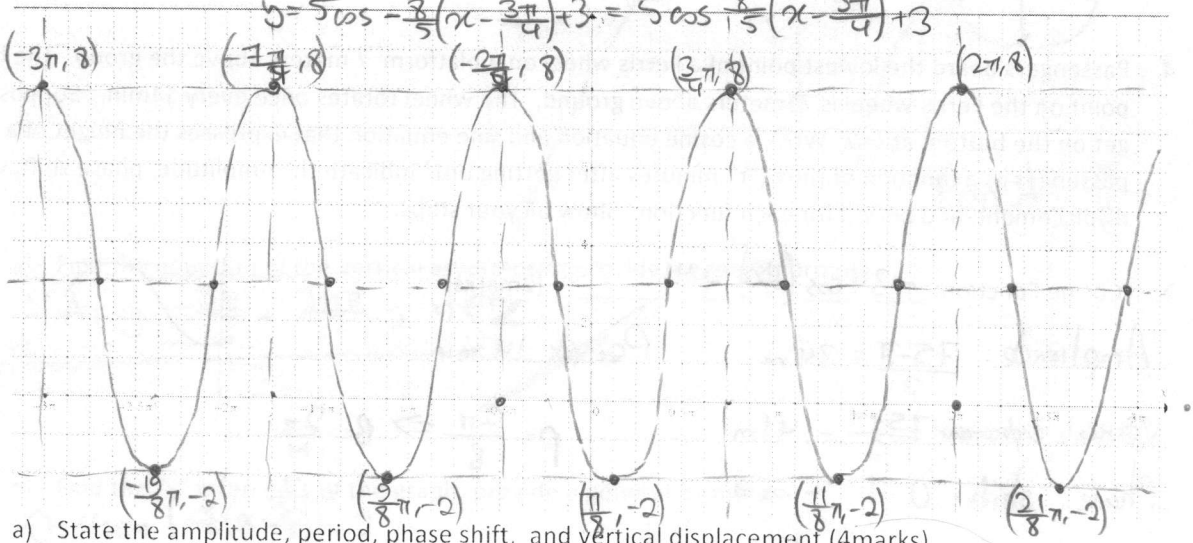
$3\theta = \sin^{-1}\left(-\frac{4}{9}\right) = 333.6°, 693.6°, 1053.6°$

$26.38°, 386.38°, 746.38°,$

$\theta = 111.2°, 231.2°, 351.2°, 8.8°, 128.8°, 248.8$

2. Graph the function : $y = 5\cos\frac{4}{5}\left(\frac{3\pi}{2} - 2x\right) + 3$ for $-3\pi \le x \le 3\pi$. (3marks)

$y = 5\cos\frac{-8}{5}\left(x - \frac{3\pi}{4}\right) + 3 = 5\cos\frac{8}{5}\left(x - \frac{3\pi}{4}\right) + 3$



a) State the amplitude, period, phase shift, and vertical displacement (4marks)

Amplitude: 5

Period: $\frac{2\pi}{\frac{8}{5}} = \frac{5\pi}{4}$

Phase shift: $\frac{3\pi}{4}$ right

Vertical displacement: 3 up.

b) Indicate the coordinates of the maximum and minimum points on the graph(2marks)

Done

c) Find the X-intercepts for $-3\pi \le x \le 3\pi$ (2marks)

$5\cos\frac{8}{5}\left(x-\frac{3\pi}{4}\right)+3=0$

$\theta = \frac{8}{5}\left(x-\frac{3\pi}{4}\right)$

$\cos\theta = -\frac{3}{5}$

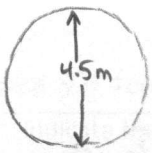
$\theta = \cos^{-1}\left(-\frac{3}{5}\right) = 2.21^R, -2.21^R$

$\theta = \frac{8}{5}\left(x-\frac{3\pi}{4}\right) \Rightarrow x = \theta + \frac{3\pi}{4} = 3.74^R, 0.972^R$

$(x,y) = (3.74^R, 0), (7.67^R, 0), (-0.19^R, 0), (-4.11^R, 0), (-8.04^R, 0)$
$(0.972^R, 0), (4.90^R, 0), (8.83^R, 0), (-2.95^R, 0)$
$(-6.88^R, 0)$

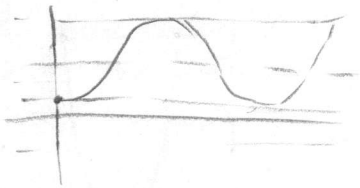
3. A vehicle with a large wheel [diameter of 4.5meters] travelled 25km. What is the angular rotation of the wheel?

$\frac{\theta}{360^\circ} = \frac{arc}{2\pi r} \Rightarrow \theta = \frac{360^\circ \times 25000\,m}{2\pi(2.25m)} = 636619.77^\circ$



4. Passengers board the lowest point of a Ferris wheel on a platform 7 meters above the group. The highest point on the Ferris wheel is 75meters above ground. The wheel rotates once every 14min.. Suppose you get on the bottom at *t=0*. Write a cosine equation and sine equation that expresses the height of a passenger as a function of time ("t" minutes after getting on). Indicate the amplitude, phase shift, vertical displacement, and period for each function. Show all your steps.

a) Cosine Function: $-34\cos\left(\frac{\pi}{7}x\right)+41$ (2marks)



Amplitude: $\frac{75-7}{2} = 34\,m$ Period: 14 min

Vertical displacement: $\frac{75+7}{2} = 41m$

Phase shift: 0 if $-\cos\theta$

$P = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{14} = \frac{\pi}{7}$

b) Sine Function: $34\sin\left[\frac{\pi}{7}(x-3.5)\right]+41$ (2marks)

Amplitude: 34m

V.D. = 41m

Phase shift: -3.5

Period: 14min

$P = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{14} = \frac{\pi}{7}$

c) Within the first 30min, at what times, "t" minutes, will a passenger on the Ferris be at least 65metres above ground? Show all your work and steps (4marks)

$34\sin\left[\frac{\pi}{7}(x-3.5)\right]+41=65$ (the bracketed part $=\theta$)

Period: 14

$\frac{\pi}{7}(x-3.5)=\theta=\sin^{-1}\left(\frac{24}{34}\right)=0.78, 2.36$

$x_1=5.25s$

$x_2=8.75s$

$5.25s \le t \le 8.75s$

$19.25s \le t \le 22.75s$

$y=4\tan\left[-\frac{4}{6}\left(x-\frac{2\pi}{3}\right)\right]+2$

5. Graph the function: $y=4\tan\frac{4}{3}\left(\frac{\pi}{3}-\frac{x}{2}\right)+2$ and then get the coordinates of x-intercepts:

Period: $\frac{\pi}{\frac{4}{6}}=\frac{6\pi}{4}=\frac{3\pi}{2}$

a) Find the equation of the vertical asymptotes, provide a general formula

V.A: $y=\frac{5\pi}{3}+\frac{3n\pi}{2}$; $n\in\mathbb{Z}$

b) Find the "x" intercepts of the graph, provide a general formula

$0=4\tan\left(-\frac{4}{6}\left(x-\frac{2\pi}{3}\right)\right)+2$ (the bracketed part $=\theta$)

$\theta=-\frac{4}{6}\left(x-\frac{2\pi}{3}\right)=\tan^{-1}\left(-\frac{1}{2}\right)=-0.46, 2.678$

$x_1=2.79+\frac{3n\pi}{2}$

$x_2=-1.9225+\frac{3n\pi}{2}$; $n\in\mathbb{Z}$

6. $\sin\theta + \frac{\sin^2\theta}{\cos\theta} + \frac{\sin^3\theta}{\cos^2\theta} + \frac{\sin^4\theta}{\cos^3\theta} + \ldots\ldots = \cos\theta$. Solve for θ

$\sin\theta\left(1 + \tan\theta + \tan^2\theta + \tan^3\theta + \cdots\right) = \cos\theta$

$\sin\theta\left(\frac{1}{1-\tan\theta}\right) = \cos\theta$ if $0 < \theta < 45°$

$1 - \tan\theta = \tan\theta$

$2\tan\theta = 1$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) = \boxed{0.46}$

θ cannot be larger than θ b/c the tanθ chart would be infinite!

$r = \tan\theta$

$a = 1$

$\frac{a(r^n - 1)}{r - 1} \xrightarrow{\text{infinite}} \frac{a}{1 - r}$

7. Solve for "x", $0 \le x \le 2\pi$: $\sin x + 2\cos x + 3\tan x = 0$

$\sin x + 2\cos x + \frac{3\sin x}{\cos x} = 0$

$\sin x\left(1 + \frac{3}{\cos x}\right) + 2\cos x = 0$

8. Graph $y = 4\cos\theta - 3\sin\theta$ without using a graphing calculator
   a) Indicate the amplitude, period, and phase shift
   b) Write the equation as a single cosine function
   c) Write the equation as a single sine function